MINISYMPOSIUM AND PIRATE SESSION ON RANDOMLY PERTURBED (HYPER)GRAPHS

ORGANIZERS: ELAD AIGNER-HOREV, DAN HEFETZ, RYAN MARTIN

Pittsburgh Session: SIAM DM22, MS6: Randomly Perturbed (Hyper)graphs

Tuesday, June 14

Time (EDT)	**Speaker**	**Title**
2:45-3:10pm	Alan Frieze	On Randomly Weighted, Randomly Perturbed Dense Graphs
3:15-3:40pm	Andrzej Dudek	Powers of Hamiltonian Cycles in Randomly Augmented Graphs

Pirate Session: Randomly Perturbed (Hyper)graphs

Friday, June 17

Time (EDT)	**Speaker**	**Title**
8:45-9:10am	Jie Han	Clique-factors in Randomly Perturbed Hypergraphs
9:15-9:40am	Shagnik Das	Schur's Theorem for Randomly Perturbed Sets
9:45-10:10am	Yury Person	Spanning F-Cycles in Random Graphs
10:15-10:40am	Andrew Treglown	Ramsey-type Results for Randomly Perturbed Graphs and Other Settings
10:45-11:10am	Michael Krivelevich	Cycle Lengths in Randomly Perturbed Graphs

Abstracts, Tuesday June 14

2:45-3:10pm: Alan Frieze, Carnegie Mellon University, USA, *On Randomly Weighted, Randomly Perturbed Dense Graphs*

Abstract: We consider the following probabilistic model of some classical combinatorial optimization problems. We are given a n-vertex graph with minimum degree cn for some absolute constant $c > 0$. We then add $o(n^2)$ random edges and then give each edge a random weight.
For minimum spanning tree, shortest path and assignment problems we give asymptotic estimates of the value of the optimum solution. For the asymmetric travelling salesperson problem, we show that Karp's patching heuristic is asymptotically optimal.

3:15-3:40pm: Andrzej Dudek, Western Michigan University, USA, *Powers of Hamiltonian Cycles in Randomly Augmented Graphs*

Abstract: It follows from the theorems of Dirac and of Komlós, Sarközy, and Szemerédi, who confirmed the Posá-Seymour conjecture, that for every $k \geq 1$ and sufficiently large n already the minimum degree $\delta(G) \geq \frac{k}{k+1}n$ for an n-vertex graph G alone suffices to ensure the existence of the k-th power of a Hamiltonian cycle. In this talk we will study the number of random edges one has to add to a graph G with minimum degree $\delta(G) \geq \left(\frac{k}{k+1} + \varepsilon\right) n$ (with $\varepsilon > 0$) in order to create an ℓ-th power of a Hamiltonian cycle, where $\ell \geq k + 1$.
This talk is based on three projects obtained together with Sylwia Antoniuk, Christian Reiher, Andrzej Ruciński and Mathias Schacht.

Abstracts, Friday June 17

8:45-9:10am: Jie Han, Beijing Institute of Technology, China, *Clique-factors in Randomly Perturbed Hypergraphs*

Abstract: Hajnal-Szemerédi theorem on clique-factors is a cornerstone in graph theory. In this talk we will present several results on minimum-degree conditions forcing clique-factors in the randomly perturbed graphs and hypergraphs, and other closely-related settings.

9:45-9:40am: Shagnik Das, National Taiwan University, Taiwan, *Schur's Theorem for Randomly Perturbed Sets*

Abstract: A set A of integers is said to be Schur if any two-colouring of A results in monochromatic x, y and z with $x + y = z$. We study the following problem: how many random integers from $[n]$ need to be added to some $A \subseteq [n]$ to ensure with high probability that the resulting set is Schur? Hu showed in 1980 that when $|A| > 4n/5$, no random integers are needed, as A is already guaranteed to be Schur. Recently, Aigner-Horev and Person showed that for any dense set of integers $A \subseteq [n]$, adding $\omega(n^{1/3})$ random integers suffices, noting that this is optimal for sets A with $|A| \leq n/2$. In this talk, we close the gap between these two results by determining how many random integers are needed to make a set $A \subseteq [n]$ Schur when $n/2 < |A| < 4n/5$. We also initiate the study of perturbing sparse sets of integets A by providing nontrivial upper and lower bounds for the number of random integers that need to be added in this case. Joint with Charlotte Knierim and Patrick Morris.

9:45-10:10am: Yury Person, Technische Universität Ilmenau, Germany, *Spanning F-Cycles in Random Graphs*

Abstract: We extend a recent argument of Kahn, Narayanan and Park about the threshold for the appearance of the square of a Hamilton cycle to other spanning structures. In particular, for any spanning graph, we give a sufficient condition under which we may determine its threshold. As an application, we find the threshold for a set of cyclically ordered copies of C_4 that span the entire vertex set, so that any two consecutive copies overlap in exactly one edge and all overlapping edges are disjoint. This answers a question of Frieze. We also determine the threshold for edge-overlapping spanning K_r-cycles. Joint with Alberto Espuny Díaz.

10:15-10:40am: Andrew Treglown, University of Birmingham, UK, *Ramsey-type Results for Randomly Perturbed Graphs and Other Settings*

Abstract: Given graphs H_1, H_2, a graph G is (H_1, H_2)-Ramsey if for every colouring of the edges of G with red and blue, there is a red copy of H_1 or a blue copy of H_2. The topic of Ramsey properties in random graphs has been well-studied, e.g., via the seminal *Random Ramsey theorem* of Rödl and Ruciński. The study of Ramsey properties of randomly perturbed graphs was initiated by Krivelevich, Sudakov and Tetali in 2006; they determined how many random edges must be added to a dense graph to ensure the resulting graph is with high probability (K_3, K_t)-Ramsey (for $t \geq 3$). They also raised the question of generalising this result to pairs of graphs other than (K_3, K_t). In this talk I will discuss various results on this problem. For example, in joint work with Shagnik Das we resolve the case when $H_1 = K_s$ and $H_2 = K_t$ where $s, t \geq 5$. I will also discuss more recent Ramsey-type results in random discrete structures.

10:45-11:10am: Michael Krivelevich, Tel Aviv University, Israel, *Cycle Lengths in Randomly Perturbed Graphs*

Abstract: We consider the following scenario: given an n vertex graph G of independence number $\alpha(G)$, plus possibly a lower bound on the minimum degree $\delta(G)$, put a set R of r random edges on top of G. Which cycle lengths can we expect to get in $G \cup R$? Is the obtained graph typically Hamiltonian or even pancyclic? Is it likely to contain a nearly spanning cycle? Joint with Elad Aigner-Horev and Dan Hefetz.