Powers of Hamilton-cycles in pattern graphs

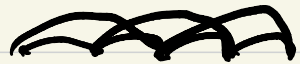
Joint work w/:

- S. Antoniuk
- Ch. Reiher
- A. Ruciński
- M. Schacht

① Degree conditions

<u>Dirac '52</u> $\delta(G) \geq \frac{n}{2} \Rightarrow G \supseteq$ Ham-cycle

<u>Tight</u>.

Q C_n^2  ?

Pósa Conj. 62 $\delta(G) \geq \frac{2}{3}n \Rightarrow G \supseteq C_n^2$

Tight

Seymour Conj $\delta(G) \geq \frac{k}{k+1}n \Rightarrow G \supseteq C_n^k$

KSS '98

H-cycle in $G(n,m)$

Pósa '76 $(k=1)$ $m \geq cn\log n \Rightarrow G(n,m)$ is Hamilton

Q $k=2$?

KO '12 $m \geq n^{\frac{3}{2}+\varepsilon} \Rightarrow G(n,m) \supseteq C_n^2$

NS '18 $m \geq cn^{3/2} \cdot \mathrm{polylog}(n)$

KNP '21 $m \geq cn^{3/2} \Rightarrow \ldots$

Q $k \geq 3$? Riordan '00 $m \geq Cn^{2-\frac{1}{k}}$, C_n^k

How to combine deg. conditions w/ random edges?

BFM '03 $\quad G, \ \delta(G) \geq \alpha n$

$$G \cup G(n, cn) \supseteq \text{Ham cycle}$$

Tight

Q $k \geq 2$?

BHKM '18 & BMPP '18

$$G, \quad \delta(G) \geq \alpha n, \quad \alpha > \frac{k}{k+1}$$

$$G \cup G(n, n^{2-\frac{1}{k+1}-\varepsilon}) \supseteq C_n^{k+1}$$

DRRS '18 $\quad \delta(G) \geq \alpha n, \quad \alpha > \frac{k}{k+1}$

$$G \cup G(n, cn) \supseteq C_n^{k+1}$$ Tight

Tight

Q $G \cup G(n, cn) \supseteq C_n^3$?

$\uparrow$

$\delta(G) \geq (\frac{1}{2}+\varepsilon)n$

ADRRS '19 & NT '19

$G \cup G(n, cn) \supseteq C_n^{2k+1}$, $\alpha > \frac{k}{k+1}$

Q $\alpha > \frac{1}{2}$

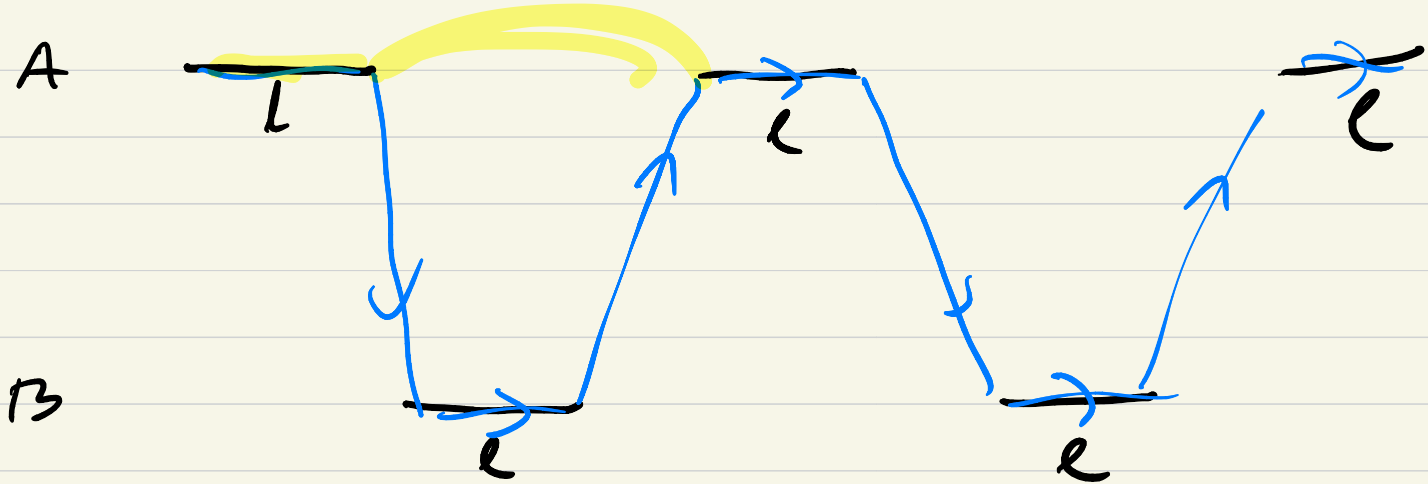
$G \cup G(n, cn) \supseteq C_n^4$?

A You need at least $n^{4/3}$ edges.

φ $(k=1)$

$G,\ \delta(G) > \left(\frac{1}{2}+\varepsilon\right)n$

$G \cup G(n, m) \supseteq C_n^p$ in $G(n, m)$



$$f_p(l) = \left[\binom{l}{2} + \binom{n-l+1}{2}\right]\frac{n}{l}$$

$$\hat{f}_p(l) = \min_{l} f_p(l)$$

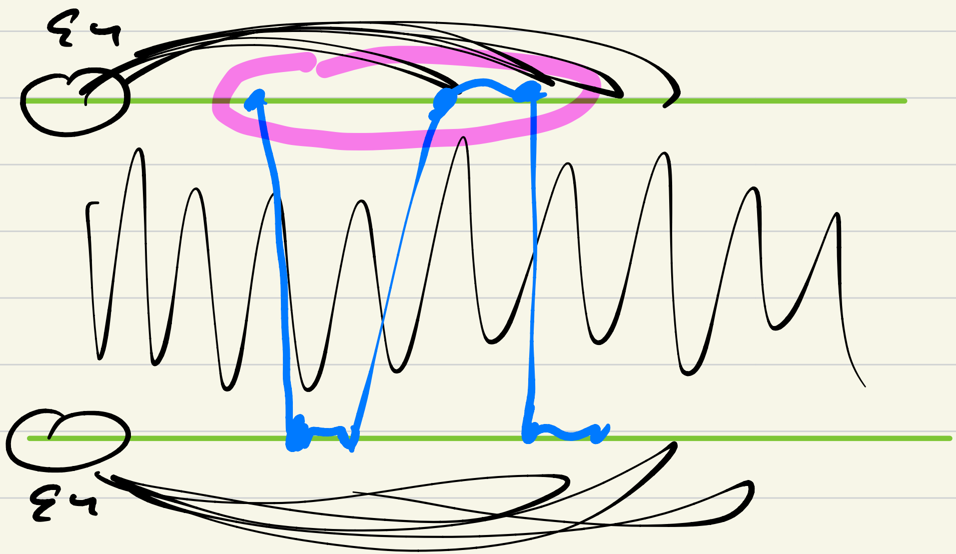
ADR, 2022 $(k=1)$, $\delta(G) \geq \left(\frac{1}{2}+\varepsilon\right)n$

$$G \cup G\left(n, n^{2-\frac{1}{f_p(\ell)}-\mu}\right) \supseteq C_n^p,$$

$$\mu = \mu(\varepsilon), \quad \mu \xrightarrow[\varepsilon \to 0]{} 0,$$

$p \geq 10.$

Open $k \geq 2$?



$$C_n^4 \supseteq \frac{n}{5} K_5$$